Beams, Beamlines and Beam Delivery

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PTCOG, Educational Workshop
May, 2007
**Beam and Ray**

- **Beam** is a Collection of Particles generated by a source.
- Trajectory of an individual particle in that beam is called a **Ray**.
- The collection of motion of the rays in a beam produce a **beam** that has an overall beam size which is modified by a beam transport system.
- However the **centroid** of the **beam** behaves like a **ray**.
- Louiville’s Theorem - Conservation of *phase space area* in the absence of external forces. *(Extra Credit)*

![Beam and Ray Diagram]
Ray Optics - Light Analogy

Light Optics / Beam Optics

Lenses
Focussing

Prism
Bending

Light Optics Uses Refraction
Beam Optics - Drift Space

- Below is a representation of the trajectory of a ray which begins with coordinates \((x_o, \theta_o)\).
- After traveling a along a drift length \(L\), winds up with coordinates \((x_f, \theta_f)\).
- The drift length is a region which applies no external forces to the beam, therefore one can write the equation which relates the final coordinates to the initial coordinates.
- In this case the transverse displacement of the ray will change according to the angle it’s trajectory makes with the reference axis.
- Since there are no outside forces acting on the particle, the transverse momentum of the ray cannot change and it’s angle remains constant. A matrix representation of that equation can also be written.

\[
\begin{align*}
\begin{bmatrix}
x_f \\
\vartheta_f
\end{bmatrix} &= 
\begin{bmatrix}
1 & L \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
x_o \\
\vartheta_o
\end{bmatrix}
\end{align*}
\]
Beam Optics - Focusing

Thin Focusing Element

• Below is a representation of the trajectory of a ray which begins with coordinates \((x_o, \theta_o)\).

• After traveling through a focusing element with focal length \(f\), winds up with coordinates \((x_f, \theta_f)\).

• Note that the drift lengths before and after the focusing element are not considered here.

• In this case, if we consider the coordinates of the ray immediately before entering the infinitesimally thin lens compared to the coordinates immediately after we know that the effect of the focusing force was to change the angle of the beam.

• Since no drift length was traversed there has been no opportunity for the transverse offset to change.

• One can now write the equation which relates the final coordinates to the initial coordinate.

\[
\begin{align*}
x_f &= x_o \\
\theta_f &= -(1/f) \cdot x_o + \theta_o
\end{align*}
\]

\[
\begin{pmatrix}
x_f \\
\theta_f
\end{pmatrix} =
\begin{pmatrix}
1 & 0 \\
-1/f & 1
\end{pmatrix}
\begin{pmatrix}
x_o \\
\theta_o
\end{pmatrix}
\]
Beam Optics - Momentum

- Thus far we have considered a 2x2 matrix, or offset and transverse momentum of a particle.
- It is also important to consider the “longitudinal” momentum of a particle and understand it’s effect a particle.
- We define $\delta = \Delta p/p$, as the fractional deviation of a particle’s momentum ($p$) from a reference momentum.
- Such a reference momentum can be the magnetic field set in a dipole that is required to bend a particle of momentum $p$ a certain desired angle.
- Rays of different momentum will bend different amounts. The transfer matrix for a wedge (Lengthless) dipole can be written as:

\[
\begin{pmatrix}
\frac{x}{x} & \frac{x}{\theta} & \frac{x}{\delta} \\
\frac{\theta}{x} & \frac{\theta}{\theta} & \frac{\theta}{\delta} \\
\frac{\delta}{x} & \frac{\delta}{\theta} & \frac{\delta}{\delta}
\end{pmatrix} = \begin{pmatrix}
1 & 0 & 0 \\
\frac{\rho}{\sin \theta} & 1 & \sin \theta \\
\frac{\rho}{\theta} & 0 & 1
\end{pmatrix}
\]

$(x/\delta) = \text{Dispersion}$ and $(\theta/\delta) = \text{Angular Dispersion}$
In this case the final transverse offset $x_f$ is independent of the initial angle $\theta_o$. In this case $(x/\theta)=0$. For the particular case above this implies that
\[ L_1 + L_2 - (L_1 L_2 / f) = 0, \text{ or } f = L_1 L_2 / (L_1 + L_2). \]

In this case the final angle $\theta_f$ is independent of the initial angle $\theta_o$. In this case $(\theta/\theta)=0$. For the particular case above this implies that
\[ 1 - (L_1 / f) = 0, \text{ or } f = L_1. \]

In this case the transverse offset $x_f$ is independent of the initial transverse offset $x_o$. In this case $(x/x)=0$. For the particular case above this implies that
\[ 1 - (L_2 / f) = 0, \text{ or } f = L_2. \]
Magnetics: Quadrupoles

\[ \mathbf{F} = q \mathbf{v} \times \mathbf{B} \]
Magnetics: Quadrupoles

\[ \vec{F} = q \vec{v} \times \vec{B} \]
Achromaticity

\[
\frac{x}{\delta} \neq 0
\]

\[
\frac{x}{\delta} = \frac{\theta}{\delta} = 0
\]

A Dipole behaves like a prism. Higher energy beams are bent less, therefore after a dipole, there is a correlation between the position and/or angle with the Ray Momentum.
Achromaticity

\[(x/\delta) \neq 0\]

\[(x/\delta) = (\theta/\delta) = 0\]

A Dipole behaves like a prism. Higher energy beams are bent less, therefore after a dipole, there is a correlation between the position and/or angle with the Ray Momentum.
Gaussian Folding in Energy Analysis

Beam Size = $\sqrt{x_0^2 + \left(\frac{x}{\delta}\right)^2} \frac{dP}{P}$

The animations above graphically illustrate the convolution of two rectangle functions (left) and two Gaussians (right). In the plots, the green curve shows the convolution of the blue and red curves as a function of $\xi$, the position indicated by the vertical green line. The gray region indicates the product as a function of $\xi$, so its area as a function of $\xi$ is precisely the convolution. mathworld.wolfram.com/Convolution.html
**Beam**

- A beam is a collection of many particles all of whose longitudinal and transverse momenta are relatively close enough to be transported through a beam transport system and remain more or less close to each other in all coordinates.

- One characterizes the transverse properties of a beam by plotting the phase space diagram below in which the transverse particle position and transverse momentum of each particle in the beam is plotted.
Distribution of Particles in “Phase Space”

Uniform

Gaussian

\[ f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2}\left[\frac{x-\mu}{\sigma}\right]^2\right) \]
**Phase Space Representation**

Due to the mechanism by which beams are produced, usually the particle distribution can be represented as a gaussian distribution. Therefore, in two dimensions, the particle number density can be represented as:

\[ \rho(x, \vartheta) = \rho_0 e^{-(\gamma x^2 + \beta \theta^2)} \]

It can be seen from this equation that the locus of constant particle distribution is \( \sqrt{\gamma x^2 + \beta \theta^2} = \text{Constant} \). Note that this is the equation of an ellipse, and this is the form normally used for the outline of the phase space. Actually this is the outline inside which is contained \(1/e\) (~65% of the particles). The area of this ellipse is \( \pi \alpha \beta \), otherwise written as \( \pi \epsilon \) where \( \epsilon \) is the ‘emittance’ of the beam. Note that when an emittance is quoted, it is important to ask what fraction of particles are included within the ellipse, it is not always \(1/e\). This is especially important when aperture restrictions are an issue.

\[ \gamma x^2 + 2\alpha_x xx' + \beta_x x'^2 = \epsilon_{x,\text{full}} \]

\[ \epsilon_{x,\text{rms}} = (\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2)^{\frac{1}{2}} \]

Phase space Area = \( \pi \epsilon \) (mm•mrad)
Evolution of Beam Phase Space

Upright Ellipse = Waist
(no correlations)
Cyclotron Extracted Beam Profiles (x)

X Profiles and Fits

- X [mm]

Legend:
- 1: #REF!
- 2: #REF!
- 3: #REF!
- 4: #REF!
- 5: #REF!
- 6: #REF!
Beam Size Plotted against Quad Strength

Waist fit plot

Beam Size Plotted against Quad Strength
What happens to the beam from the last magnet to Isocenter?
**Beam Size, 3 m drift, From Gantry to Isocenter**

- **Emit = 18 mm mrad, sigma = 3 mm**
  - Typical Cyclotron Degraded Beam

- **Emit = 18, sigma = 9 mm**

- **Emit = 5, sigma = 3 mm**
  - Typical Synchrotron Beam

\[ X_{	ext{DIP}}^2 \sim X_{	ext{ISO}}^2 + L^2\theta_{	ext{ISO}}^2 \]
Effect of Beam Size on Gantry Dipole and Power

PS Current $\sim$ Gap (2.1 sigma)
Magnet weight $\sim$ gap $^2$
Power $\sim$ Current $^2$ $\sim$ gap $^2$

$$\nu(t) = L \frac{dI(t)}{dt}$$

For Larger Emittance Beams, the power and weight requirements increase a factor of 3 when reducing the beam size from 6mm to 3mm.

Reducing emittance means reducing the current intensity.

Even smaller optical beam size is necessary when considering scatter.
Multiple Scattering

\[ \theta_f = \sqrt{\theta_i^2 + \theta_{MS}^2} \]
Beamline Example:
Beamline Tuning / Steering

Adjust or Predictable?
Beam Delivery: Double Scattering

From a Beam Point of view:

Step 1

Step 2

Step 3
Step 1 - First Scatterer

X - X'

X - Y

[Diagram showing scatter plot analysis]
Step 2 - Drift and Post

$X - X'$

$X - Y$
Step 3 - Second Scatterer and Drift
Beam Delivery: Scanning

• Simpler?
  – No beam modifying devices ???
• More Difficult?
  – Control the beam - no safety net
  – Many issues to consider
    • *Beam Shape*
    • *Beam Size*
    • *Beam Trajectory*
    • *Materials in the beam path*

Scanning relies on the SUPERPOSITION of Unscattered beams
**Beam Shape**: Superimposing Gaussians

- **Gaussians** have a nice property when adding them together in a Voxel type manner.
Beamshape: Monte Carlo Studies of Aberration Effects and Correction

Don’t assume beam is always Gaussian?
Beam Shape: How to IMPLEMENT a small penumbra at the edge and conform to the desired dose distribution inside the target?

Use of Beam Edge in Scanning?

- There are so many disadvantages to a small beam
  - Tolerances (e.g. 1mm/40cm = .25%)
  - Time for Scanning
  - Safety/Dosimetry
  - Hard to Make

- What are the Requirements?
  - Small Beam?
  - Or sharp edge?

- Is a sharp edge the same as a small beam?
A Sharp Edged non-gaussian beam

- This is a sharp Edged beam.
- Disadvantages in a type of scanning.
  - Tighter Tolerances when adding them together in a Voxel type scanning Scheme Of in the line direction of continuous scanning

This has less sensitivity to periodic addition, but is still sharper
Phase space representation of an Aperture

One way to make a beam edge sharper is by use of an aperture.
Beam Shape: Apertures
Hard to keep an edge

Initial Rectangular Beam

2.5 m Drift in Vacuum

Drift

Penumbra

4.5 cm of Water

Multiple Scattering

18 cm of Water
Beam Size/Shape:
How can one achieve a sharp edge beam Without a collimator right at the patient?

- With a *selectable effective drift* one can also control the ‘penumbra’ which could be useful in matching.
PSI Gantry 2

The Ultimate IMPT Device?
– Or The Ultimate IMPT R&D Device?
– Upstream Instrumentation minimized (1)
– Vacuum System (1 & 2)
– Beam size ~ 3mm (1 & 2)
– Moderate Field Size
  • 20x PPS (1), 20cmx12cm (2)
– Infinite SAD (1 &2)
  • (Adjustable?)
– EDGE CONTROL !!! (2)
Beam Size:

Materials in the Nozzle Beam Path

• Instrumentation (Two points make a straight line; \((x, p_x, y, p_y)\) - One point implies you have to be confident that the angle is right! (Commissioning/QA)

• Gas (Air, Vacuum (+ Vacuum Window), Helium)

• Windows
  – Gantry Dipole
  – Helium Chamber Windows
  – End of one Vacuum System

• Range Shifter/Ridge Filter
Field Size vs. Beam Size
due to Window Thickness

Window thickness depends upon minimum foil dimension.
Foil dimension is determined by field size.

\[ \theta_0 = \frac{13.6 \text{ MeV}}{\beta c \rho} \approx \sqrt{x/X_0} \left[ 1 + 0.038 \ln(x/X_0) \right] \]
Homework:

- What are the requirements for the beam size/shape for scanning? (e.g. Tradeoffs)
- What time structure are which instruments capable of monitoring?
- What will it take for ‘industry’, to implement the ‘innovations’ that we have been discussing?
- Now that you’ve heard this discussion, is it harder or easier to make a beam, than you thought it was?
End

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Beams

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